1. Field strength **(3 pts)**

Calculate the amplitudes of E and B at a distance of 50 cm away from an isotropic, monochromatic point source that radiates at a constant power level of 100 W.

$I = P/(4\pi r^2) = 100\ \text{W}/(3.14 \cdot 1\ \text{m}^2) = 31.8\ \text{W/m}^2$

$$E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}} = \sqrt{\frac{63.6\ \text{VA/m}^2}{8.85\cdot 10^{-12}\ \text{As/Vm}\cdot 3\cdot 10^8\ \text{m/s}}} = 155\ \text{V/m}\ ;\ B_0 = E_0/c = 5.2\cdot 10^{-7}\ \text{Vs}$$

2. Sellmeier equation **(5 pts)**

Show that Cauchy's equation, $n = C_1 + C_2/\lambda^2 + C_3/\lambda^4 + \ldots$, is an approximation of Sellmeier's equation,

$$n^2 = 1 + \sum_j \frac{A_j\lambda^2}{\lambda^2 - \lambda_{0,j}^2}$$

when $\lambda >> \lambda_{0,j}$ (where $\lambda_{0,j}$ are the wavelengths associated with the relaxation frequencies $\nu_{0,j}$).

Squaring the Cauchy form yields

$$n^2 = \left(C_1 + C_2\lambda^2 + \ldots\right)^2 = C_1^2 + 2\frac{C_1C_2}{\lambda^2} + \mathrm{O}(\lambda^4)$$

On the other hand, expansion of the Sellmeier formula gives

$$n^2 = 1 + \frac{A}{1-\lambda_0^2/\lambda^2} \approx 1 + A\left(1+\lambda_0^2/\lambda^2\right) = (1+A) + \frac{A\lambda_0^2}{\lambda^2}$$

(One could do that for all members of the sum, but that's not necessary for the general identity.) Then, equating the coefficients of the two forms yields,

$$C_1 = (1+A)^{1/2};\quad C_2 = \frac{A\lambda_0^2}{2C_1} = \frac{A\lambda_0^2}{2(1+A)^{1/2}}$$

3. Rayleigh scattering **(2 pts)**

Determine the relative amount of scattering of a dilute gas mixture for the λ = 580 nm and λ = 400 nm components of white light. Implications?

4.2 The degree of Rayleigh scattering is proportional to $1/\lambda^4$. But $\lambda_y = 1.45\lambda_v$ and so $1/\lambda_y^4 = (1/1.45\lambda_v)^4$ hence violet is scattered $(1.45)^4 = 4.42$ times more intensely than yellow. The ratio of yellow to violet is 22.6%.

4. Damped driven oscillator **(5 pts)**

In the equation for the damped driven oscillator, $m_e\ddot{x} + m_e\gamma\dot{x} + m_e\omega_0^2 x = q_e E(t)$, explain the significance of each term and show that the amplitude x_0 of a harmonic solution depends on the field amplitude, E_0, as

$$x_0 = \frac{q_e E_0}{m_e} \frac{1}{\left[\left(\omega_0^2 - \omega^2\right) + \gamma^2\omega^2\right]^{1/2}}$$

How does the phase lag α depend on ω?

4.4 (a) On the left-hand side are the inertial, drag force, and elastic force terms; on the right-hand side is the electric driving force. (b) $x_0(-\omega^2 + \omega_0^2 + i\gamma\omega) = (q_e E_0/m_e)\exp(i\alpha)$, forming the absolute square of both sides yields $x_0^2[(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2] = (q_e E_0/m_e)^2$ and x_0 follows by division and taking the square root. (c) As for α, divide the imaginary parts of both sides of the first equation above, namely $x_0\gamma\omega = (q_e E_0/m_e)\sin\alpha$, by the real parts, $x_0(\omega_0^2 - \omega^2) = (q_e E_0/m_e)\cos\alpha$ to obtain $\alpha = \tan^{-1}[\gamma\omega/(\omega_0^2 - \omega^2)]$. α ranges continuously from 0 to $\pi/2$ to π.

5. Wave in thin glass plate **(5 pts)**

Show that an observer sees the field of a light beam, $E_u = E_0 \exp(i\omega[t - y/c])$ in vacuum, change to $E_p = E_0 \exp(i\omega[t - (n-1)\Delta y/c - y/c])$ in an absorption-free glass plate of index n and thickness Δy. Start with the no. of oscillations of the wave field within Δy in vacuum and in the medium. Then show that for $n \approx 1$, or if Δy is very small, $E_p = E_u + \omega(n-1)\Delta y/c \cdot E_u \exp(-i\pi/2)$. This result shows that the secondary wave in a medium is phase-shifted by 90° ($\alpha = \pi/2$) against the primary wave, even as $\Delta y \to 0$.

4.5 (a) The phase angle is retarded by an amount $(n\Delta y 2\pi/\lambda) - \Delta y 2\pi/\lambda$ or $(n-1)\Delta y\omega/c$. Thus $E_p = E_0 \exp i\omega[t - (n-1)\Delta y/c - y/c]$ or $E_p = E_0 \exp[-i\omega(n-1)\Delta y/c] \exp i\omega(t - y/c)$ (b) Since $e^x \approx 1 + x$ for small x, if $n \approx 1$ or $\Delta y \ll 1$, $\exp[-i\omega(n-1)\Delta y/c] \approx 1 - i\omega(n-1)\Delta y/c$ and since $\exp(-i\pi/2) = -i$, $E_p = E_u + \omega(n-1)\Delta y(E_u/c)\exp(-i\pi/2)$.